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LETTER TO THE EDITOR

On the fitness of the sparse-matrix technique applied to the self-consistent GTD formulation

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Abstract. This letter points out that it can be very computationally efficient to apply the sparse-matrix storage technique and the Gauss-Seidel iterative method to the self-consistent GTD (Geometrical Theory of Diffraction) formulation.

The self-consistent GTD formulation [1] is a simple approximate method for treating the two-dimensional diffraction problem of a perfectly conducting convex cylinder with arbitrary cross section. It states that the convex surface may be alternatively modelled as flat planes linked with a finite number of edges; consequently, the original convex surface diffraction problem is transformed into one of just considering the edge diffraction and geometrical optic field calculation.

The self-consistent GTD formulation always results in a sparse matrix. To explain this point, the two-dimensional N-sided polygonal cylinder in figure 1 may serve as an example, in which the elements for the nth corner and their relative positions in

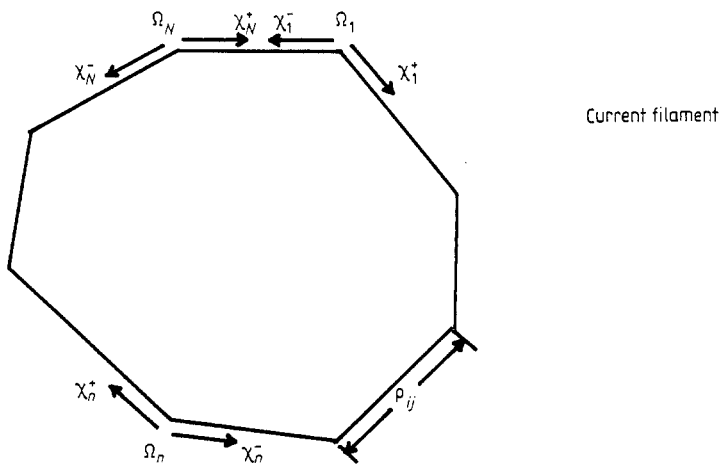

Figure 1. An N-sided polygonal cylinder (with N corners) based on the self-consistent GTD formulation. ρ_{ij} is the width of the linking flat plane between the vicinal corners Ω_i and Ω_j. The $\chi_i^{\mp}$ are the unknown surface field coefficients associated with the corner Ω_i.

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the matrix of self-consistent GTD formulation are as shown below:

Column	$2n-3$	$2n-2$	$2n-1$	$2n$	$2n+1$	$2n+2$
Row $2n-1$	$-T_{n,n-1}$	0	1	0	0	$-R_{n,n+1}$
Row $2n$	$-R_{n,n-1}$	0	0	1	0	$-T_{n,n+1}$

$(n=2,3,\ldots,N-1)$

where $T_{m,n}$ (transmission coefficient) and $R_{m,n}$ (reflection coefficient) both are functions of the Kouyoumjian scalar edge-diffraction coefficients [2]; furthermore, the subscripts m and n are the indices of target corner and source corner respectively. The elements for the corners 1 and N can be similarly calculated and set into their relative positions in the matrix as follows:

Columns

	1	2						2N	
Row 1	1	0	0	$-R_{1,2}$	$\cdots$		$-T_{1,N}$	0	corner 1 data
	0	1	0	$-T_{1,2}$			$-R_{1,N}$	0	
		$\vdots$			$\ddots$				
Row 2N	0	$-R_{N,1}$			$-T_{N,N-1}$	0	1	0	corner N data
	0	$-T_{N,1}$			$-R_{N,N-1}$	0	0	1	

It may be clearly observed that this matrix has a sparse appearance. There are only three entries in each row; in other words, there is only a fraction $3/2N$ of non-zero elements in this matrix for an N-sided polygonal model. For larger cylinders, larger N is always needed, which makes this fraction smaller. Taking an example, for a circular cylinder of radius $a=4\lambda$ (where λ is the operating wavelength in free space) and the flat planes are equally set to be 0.931λ ($N=27$), the corresponding sparseness is only 5.5556%.

To employ the sparse-matrix technique, it is necessary to select a proper representation for the matrix under consideration. The sparse row-wise format [3], which is a popular and simple storage scheme for sparse matrices, is proposed because it is a good match for our application when the Gauss-Seidel iterative method is used to solve the self-consistent GTD formulation. The 'Row-wise representation complete and ordered' (RR(C)O), in which the non-zero elements of each row are stored in the ascending order of their column indices, is further preferred in this study.

It is well known that, for solving a linear system $\bar{A}\cdot\bar{X}=\bar{B}$ in most cases at least, the application of iterative methods has an advantage of operation saving over the direct method [4], since it may only deal with the non-zero elements in the matrix of the coefficients of the linear equation system. The Gauss-Seidel method can be taken as an example, in which the sufficient condition for convergence is

$$|a_{mm}| > \sum_{j=1,j\neq m}^{M} |a_{mj}| \qquad \begin{matrix} m=1,2,\ldots,M \\ \text{general } M\times M \text{ matrix} \end{matrix}$$

where the a_{ij} are the elements in the sparse matrix. The detailed proof of this point is given in [5]. Then, since all the diagonal elements are 1s in the matrix of the present self-consistent GTD formulation, the key problem to check whether this algorithm is suitable or not for this study is alternatively to examine the following conditions:

$$\begin{array}{llll} |R_{1,2}|+|T_{1,N}| & \text{and} & |T_{1,2}|+|R_{1,N}|<1 & \text{for the 1st edge} \\ |T_{n,n-1}|+|R_{n,n+1}| & \text{and} & |R_{n,n-1}|+|T_{n,n+1}|<1 & \text{for the } n\text{th edge} \\ |R_{N,1}|+|T_{N,N-1}| & \text{and} & |T_{N,1}|+|R_{N,N-1}|<1 & \text{for the } N\text{th edge.} \end{array} \qquad (1)$$

For further discussion, let us write out the transmission coefficient T_{ij} and reflection coefficient R_{ij} as

$$T_{ij} = \frac{\exp(-jk\rho_{ij})}{\sqrt{\rho_{ij}}} \frac{1}{2} D_{s,h}(360° - \alpha_i, 0°, 90°)$$

$$R_{ij} = \frac{\exp(-jk\rho_{ij})}{\sqrt{\rho_{ij}}} \frac{1}{2} D_{s,h}(0°, 0°, 90°)$$

where ρ_{ij} is the distance between the two adjacent corners Ω_i and Ω_j, α_i is the edge internal angle for corner Ω_i, and $D_{s,h}(\varphi, \varphi', \beta_0)$ is the scalar edge-diffraction coefficient mentioned above. It can be observed that whether or not those diagonally dominant conditions stated in (1) are satisfied is strongly dependent on the choice of the flat plane's width ρ_{ij}'s between the vicinal corners Ω_i and Ω_j. The larger ρ_{ij}, the easier it is to satisfy the conditions as long as the polygonal cylinder approximation is still good for the original convex cylinder in the diffraction calculation. On the other hand, the ρ_{ij} should not be set too small so as to make sure that, for any certain corner on the polygon, it is located at the far field of the neighbouring corners. This minimum criterion can be estimated to be 0.7λ [6]. Under this requirement and because of the numerical range of the diffraction coefficients, those convergence conditions stated above are very easily satisfied. Offering a numerical example may be helpful to accept this point, and to do this work the FORTRAN subroutine given in [7] is useful in calculating the diffraction coefficients $D_{s,h}$. For a circular cylinder (radius $a = 1.5\lambda$) being modelled by a 12-sided polygonal cylinder with equal ρ_{ij} being 0.776λ, if the H-polarisation incidence is considered, then the following parameters result:

$$|T_{ij}| = 0.44066 \qquad |R_{ij}| = 0.03462.$$

We have $|T_{ij}| + |R_{ij}| = 0.47528 < 1$.

Conclusively, the self-consistent GTD formulation is well suited to the Gauss-Seidel iterative algorithm.

It is further worth noting that, since all the diagonal elements are 1, it is not necessary to store them in the RR(C)O format. This will not affect the Gauss-Seidel iterative algorithm when setting the factor a_{mm}^{-1} to 1 in the iterative equation.

Although the establishment of storing the matrix coefficients by using the proposed format needs some time-consuming effort, those coefficients are only dependent on the scatterer itself and the wave polarisation, and have nothing to do with the source positions. In most practical situations, when people study the diffraction problems, they are usually given a specific scatterer (say, a cylinder) and are required to find the corresponding effects whenever the position of the source is changed. The location of a proper position of a missile's antenna may serve as an example to demonstrate this point. Therefore, the work of tabulating the RR(C)O data need only be done once in most common cases when the self-consistent GTD approach is adopted. That is, the source-disturbing data will modify the excitation column vector only and need not be stored in the row-wise storage format.

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